Exact solutions in two–dimensional string cosmology with back reaction

Sukanta Bose * and Sayan Kar †

Inter–University Centre for Astronomy and Astrophysics,

Post Bag 4, Ganeshkhind, Pune, 411 007, INDIA

Abstract

We present analytic cosmological solutions in a model of two-dimensional dilaton gravity with back reaction. One of these solutions exhibits a graceful exit from the inflationary to the FRW phase and is nonsingular everywhere. A duality related second solution is found to exist only in the "pre-big-bang" epoch and is singular at $\tau = 0$. In either case back reaction is shown to play a crucial role in determining the specific nature of these geometries.

Pacs: 98.80.Cq, 04.60.Kz, 11.25.-w

Typeset using REVTEX

---

*Electronic Address : sbose@iucaa.ernet.in

†Electronic Address : sayan@iucaa.ernet.in

Two-dimensional (2D) models of gravity have acquired a special status over the last few years. In spite of being largely pedagogical, they form a useful set where the usual problems of four-dimensional (4D) theories of gravity such as black–hole evaporation, cosmological singularities, etc. can be addressed and answered with reasonable confidence. Thus, one expects that the physical insights obtained in such a 2D setting would carry over to some extent to the actual 4D scenario, although the technicalities will be more involved there.

Among such models, the one that has been investigated quite thoroughly, is due to Callan, Giddings, Harvey, and Strominger (CGHS) [1]. The CGHS model is largely inspired by target space, low–energy effective string theory compactified down to two dimensions. The field content of this model constitutes the dilaton (ϕ), the graviton ($g_{\mu\nu}$), and a multi-component conformally coupled massless scalar matter field denoted by f_i ($i = 1, 2, ...N$). The CGHS action is

$$S_0 = \frac{1}{2\pi} \int d^2x \sqrt{-g} \left\{ e^{-2\phi} \left[R + 4 \left(\nabla \phi \right)^2 - 4\Lambda \right] - \frac{1}{2} \sum_{i=1}^{N} \left(\nabla f_i \right)^2 \right\} \ , \tag{1}$$

where Λ is a cosmological constant term, which will henceforth be set equal to zero.

The cosmological solutions to the above 2D action were first studied by Mazzitelli and Russo [2]. However, it is the recent work of Rey [3] that has revived interest in this model. Rey showed that the above action has classical solutions with features similar to the cosmological solutions in 4D superstring theory (for reviews, see Refs. [4]). These solutions are characterised by two branches separated in time by a curvature singularity. The first branch describes superinflationary behavior, with accelerated expansion and monotonically increasing scalar curvature, whereas the second branch describes an FRW universe. Although the existence of a superinflationary phase in the early history of the universe is promising, the full solution is plagued by the lack of a smooth singularity-free transition from this "pre-big-bang" phase to the FRW phase. This is known as the "graceful exit" problem of superstring cosmology.

It was also shown by Rey that incorporating back reaction effects cures the graceful exit problem. (For one approach towards a solution to the back reaction problem in two

dimensions, see Ref. [5].) However, his cure depended on the restrictive requirement that the total number, N, of massless scalar matter fields in the theory be less than twenty four. Subsequently, there have been some attempts at removing this barrier by modifying the 2D classical action and studying the back reaction effects on corresponding solutions [6,7]. However, in these papers, in cases where the restriction on N is removed, one finds that, unlike the CGHS action, the corresponding 2D classical actions cannot be motivated by reduction from a realistic 4D action.

It was shown in [8] that one can remove the barrier on N in Rey's model without modifying the tree level action (1). As in Rey, the main idea in [8] was to study back reaction effects by incorporating one-loop correction terms in the CGHS action. However, since these one-loop terms are well defined only up to the addition of local covariant counterterms [10], different combinations of such terms lead to different models. One chooses to analyse only those models that display the physical phenomena of interest. The counterterms used in the one-loop action of [8] are different from those used by Rey (which were the ones corresponding to the RST model [9]). The resulting quantum-corrected cosmological solutions of [8] are similar to Rey's, with the added advantage that the graceful exit problem is solved for any positive value of N.

It is important to note that only the asymptotic behavior of the quantum-corrected solutions were studied in Refs. [3] and [8]. In this paper, we find the exact solutions of these models and discuss their physical features. In particular, we address the following two issues. We show that in our quantum-corrected solutions, where the graceful exit problem is solved, the Weak Energy Condition is violated. We also show that the scale factor duality is not respected by the exact solutions. However, there exists a duality transformation that relates one solution to another.

The one-loop corrected 2D action studied in Ref. [8] is

$$S_1 = S_0 + \frac{N\hbar}{24\pi} \int d^2x \sqrt{-g} \left(-\frac{1}{4} R \Box^{-1} R + 2(\nabla\phi)^2 - 3\phi R \right) \ , \tag{2}$$

where $\Box_x G(x, x') = \delta^2(x - x')/\sqrt{-g(x)}$. The first term in the parenthesis is the Polyakov-

Liouville term that reproduces the trace anomaly for massless scalar fields [1,13]. The remaining terms in the parenthesis are local covariant counterterms that differ from the ones occurring in the RST model. The higher order corrections beyond one loop are dropped by using the large N approximation where $N \to \infty$ as $\hbar \to 0$ such that $\kappa \equiv N\hbar/12$ remains finite. The metric components in the conformal gauge, $g_{\mu\nu} = e^{2\rho}\eta_{\mu\nu}$, when expressed in the double null coordinates, $x^{\pm} \equiv t \pm x$, are $g_{+-} = -e^{2\rho}/2$ and $g_{\pm\pm} = 0$.

We now use the following redefined fields

$$\Sigma \equiv (\phi - \rho), \qquad \Phi \equiv e^{-2\phi} - \kappa\phi + \frac{\kappa}{2}\rho = e^{-2\phi} - \frac{\kappa}{2}\rho - \kappa\Sigma \ . \tag{3}$$

In terms of these fields the one-loop corrected action in the conformal gauge simplifies to

$$S_1 = \frac{1}{\pi} \int d^2x \left[\partial_+ \Phi \partial_- \Sigma + \partial_- \Phi \partial_+ \Sigma + \frac{1}{2} \sum_{i=1}^{N} \partial_+ f_i \partial_- f_i \right] \ , \tag{4}$$

where we have set $\Lambda = 0$. For homogeneous cosmologies with constant f_i's, the equations of motion following from the variation of the above action are

$$\frac{d^2\Phi}{dt^2} = 0 = \frac{d^2\Sigma}{dt^2} \ . \tag{5}$$

The accompanying constraints (obtained from varying S_1 in (2) with respect to $g^{\mu\nu}$ and setting $\mu = \nu = \pm$) are

$$\partial_{\pm}^2 \Phi + 2\partial_{\pm}\Phi\partial_{\pm}\Sigma = \frac{3}{2}\kappa \left[\partial_{\pm}^2 \phi - 2\partial_{\pm}\rho\partial_{\pm}\phi \right] + \kappa t_{\pm}(x^{\pm}) \ , \tag{6}$$

where $t_{\pm}(x^{\pm})$ are nonlocal functions that arise from the homogeneous part of the Green function (see [1,11]). The choice of these nonlocal functions determine the quantum state of the matter fields in the spacetime. The total matter stress tensor can be expanded in orders of $\hbar$ as $T_{\mu\nu}^f = (T_{\mu\nu}^f)_{cl} + \langle T_{\mu\nu} \rangle$, where $(T_{\pm\pm}^f)_{cl} \equiv \frac{1}{2}\sum_{i=1}^{N}(\partial_{\pm}f_i)^2$ is the classical part and $\langle T_{\pm\pm} \rangle = \kappa[\partial_{\pm}^2\rho - (\partial_{\pm}\rho)^2 - t_{\pm}(x^{\pm})]$ is the one-loop contribution [12,13]. We will choose the quantum state of the matter fields to be defined by

$$t_{\pm}(x^{\pm}) = -\frac{3}{2}\left[\partial_{\pm}^2\phi - 2\partial_{\pm}\rho\partial_{\pm}\phi \right] \ , \tag{7}$$

4

which simplifies the constraint equations for the homogeneous cosmologies to

$$\frac{d^2\Phi}{dt^2} + 2\frac{d\Phi}{dt}\frac{d\Sigma}{dt} = 0 \,. \tag{8}$$

The equations of motion (5) and the above constraint have been solved to obtain the past and future asymptotic behavior of cosmological solutions [8].

To obtain the exact expression for the scale factor a in these solutions, we define $a(t) \equiv \ln \rho(t)$ where, as defined above, t is the conformal time. We then rewrite the field equations and the constraint in terms of $\phi(\tau)$ and $a(\tau)$, where τ is defined to be the comoving time obeying $d\tau/a(\tau) \equiv dt$. They turn out to be:

$$\ddot{\phi} - \frac{\ddot{a}}{a} + \frac{\dot{a}}{a}\dot{\phi} = 0 \,, \tag{9}$$

$$2e^{-2\phi}\left(2\dot{\phi}^2 - \ddot{\phi} - \frac{\dot{a}}{a}\dot{\phi}\right) + \frac{1}{2}\kappa\left(-2\frac{\dot{a}}{a}\dot{\phi} + \frac{\ddot{a}}{a} - 2\ddot{\phi}\right) = 0 \,, \tag{10}$$

$$\left(-2e^{-2\phi}\dot{\phi} - \kappa\dot{\phi} + \frac{\kappa}{2}\frac{\dot{a}}{a}\right)\left(\dot{\phi} - \frac{\dot{a}}{a}\right) = 0 \,, \tag{11}$$

where the overdot denotes $d/d\tau$. The first two equations in this set are the equations of motion and the third is the constraint.

Note that in the classical case ($\kappa = 0$), the above equations are invariant under the scale factor duality (SFD) transformations:

$$\phi \to \phi - \ln a \qquad , \qquad a \to 1/a \,. \tag{12}$$

In terms of the redefined fields (3) (with $\kappa = 0$), this duality is essentially related to the invariance of the set of equations (5) and (8) under the transformation $\Phi \to \Sigma$ and $\Sigma \to \Phi$. Since such a transformation leaves the quantum equations ($\kappa \neq 0$) invariant as well, there should exist a duality transformation analogous to (12) for the one-loop corrected equations. We find that Eqs. (5) and (8) are invariant under the dual transformation:

$$\phi \to \phi \qquad , \qquad \ln a(t) \to -\frac{2}{\kappa}e^{-2\phi} - \ln a(t) + 3\phi \,, \tag{13}$$

5

which is equivalent to the interchange of Φ and $2\Sigma/\kappa$. Such a symmetry can be exploited to construct both classical as well as quantum solutions. A similar symmetry ($\phi \rightarrow \phi$, $\ln a(t) \rightarrow -\frac{2}{\kappa}e^{-2\phi} - \ln a(t) + \phi$) exists in Rey's model as well.

We now obtain the solutions in our one-loop corrected model. The constraint (11) contains two factors. We first choose to solve it by setting

$$\dot{\phi} = \frac{\dot{a}}{a} \qquad \text{or} \qquad \phi = \ln a \, . \tag{14}$$

Note that the first equation (9) is also solved by this condition on ϕ. Substituting the expressions for ϕ and its derivatives into (10), we obtain the following equation for the scale factor $a(\tau)$:

$$\frac{2}{a^2}\left[2\left(\frac{\dot{a}}{a}\right)^2 - \frac{\ddot{a}}{a}\right] - \frac{\kappa}{2}\frac{\ddot{a}}{a} = 0 \, . \tag{15}$$

Note that for a large the first term is small and we get a solution that is linear in τ. For a small, on the other hand, it is the first term that dominates and we obtain the superinflationary epoch in the asymptotic past, where the scale factor exhibits an inverse power-law behavior in τ. Thus, scale factor duality symmetry is preserved only in the asymptotic past and future.

We solve Eq. (15) by rewriting it as:

$$\frac{d^2}{d\tau^2}\left(-\frac{1}{a} + \frac{\kappa}{4}a\right) = 0 \; , \tag{16}$$

which yields

$$a(\tau) = \frac{2}{\kappa}\left(\alpha\tau + \beta \pm \sqrt{(\alpha\tau + \beta)^2 + \kappa}\right) \; , \tag{17}$$

where α and β are integration constants, and κ is positive. The solution with the negative sign for the square root has to be discarded because it gives a negative a. One can check that in the asymptotic past and future, the scale factor is proportional to $-\frac{1}{\tau}$ and τ, respectively. Hence, this solution exhibits scale factor duality asymptotically. The function (with the $+$ sign for the square root) is plotted in Fig. 1. Solution (17) is the quantum analogue of the first branch solution [3] in the classical case.

The important feature of this solution is that κ plays a crucial role in it. Without a finite value of κ one would end up with a scale factor that either vanishes or diverges for a finite value of τ. This would preclude a solution to either the graceful exit problem or the singularity problem.

The Ricci scalar for the above geometry turns out to be:

$$R = \frac{2\alpha^2 \kappa}{\left\{ (\alpha\tau + \beta)^2 + \kappa \right\}^{\frac{3}{2}} \left\{ \alpha\tau + \beta + \sqrt{(\alpha\tau + \beta)^2 + \kappa} \right\}} \quad , \tag{18}$$

which is everywhere finite, clearly due to a finite value of κ.

It is instructive to construct the exact solution of the field equations and constraints for the model due to Rey [3]. Following the steps similar to Eqs. (9)-(11), we find the solution to be:

$$a(\tau) = \frac{2}{\kappa} \left\{ -(\alpha\tau + \beta) \pm \sqrt{(\alpha\tau + \beta)^2 - \kappa} \right\} . \tag{19}$$

However, here it is necessary to consider the negative sign for the square root and also assume κ to be negative. Substituting $\kappa = -|\kappa|$ we get,

$$a(\tau) = \frac{2}{|\kappa|} \left\{ (\alpha\tau + \beta) + \sqrt{(\alpha\tau + \beta)^2 + |\kappa|} \right\} \quad , \tag{20}$$

which is exactly the same as the solution for the previous model except that we need to choose κ to be negative. In fact, it is also easy to see that here the geometry is nonsingular only if κ is negative. In the above solutions for both these models, note that the dilaton field coupling, e^ϕ, is actually equal to the scale factor a and, therefore, remains finite except for $\tau \to \infty$ (see Fig. 1).

Equations (17) and (20) for the scale factor in the first branch show that SFD is not respected in the exact solutions of the quantum-corrected models of Refs. [3] and [8], although it is exhibited asymptotically as $\tau \to \pm\infty$. Even without explicitly solving the equations for $a(\tau)$ one could deduce qualitatively the nature of the solutions from them. Probable solutions with a graceful exit can be categorized into three classes: (i) $a(\tau)$ is a

monotonic function, (ii) $a(\tau)$ has atleast a pair of extrema (a maximum and a minimum) in the neighborhood of $\tau = 0$, (iii) $a(\tau)$ has inflection points but no extrema.

For case (ii), with $\dot{a} = 0$ at the extrema, we must have from (16),

$$- \frac{\ddot{a}(\tau_0)}{a_0} \left[\frac{2}{a_0^2} + \frac{\kappa}{2} \right] = 0 \,. \tag{21}$$

at the extrema. Note that with κ replaced by $|\kappa|$, we get the corresponding condition for Rey's model. In the classical case ($\kappa = 0$), the above equation can be satisfied if either $a_0 \to \infty$ or $\ddot{a}(\tau_0) = 0$. The latter condition would imply a vanishing scalar curvature for a finite a_0. The classical equations of motion in fact yield two solutions, related by SFD, such that the first branch (superinflationary phase) satisfies $a_0 \to \infty$ (as $\tau_0 \to 0$), and the second branch (FRW phase) has $\ddot{a}(\tau_0) = 0$. Conditions (i) and (iii) are not realised in the classical case. Thus SFD precludes a solution to the graceful exit problem in the classical case.

In the quantum case $\kappa \neq 0$, and the equations of motion show that SFD is lost (although the duality transformation (13) exists that allows one to generate the second branch solution from the first). For case (ii) we need a_0^2 to be negative, which is physically meaningless. Cases (i) and (iii) can not be ruled out by qualitative reasoning. The exact solutions however exhibit that the scale factor in the first branch is a function satisfying (i) with asymptotic SFD. Hence, the first branch itself is an interesting cosmological solution without the graceful exit problem.

The constraint equation (11) (and a corresponding equation for Rey's model) contains a couple of factors, one of which is common to both the models, namely, $(\dot{\phi} - \dot{a}/a)$. In the previous analysis, we have set this factor to zero and therefore the corresponding solutions in the two models are the same, modulo the sign of κ. We now investigate the consequences of setting the other factor to zero in each model.

For the model defined in (2), the constraint equation (11) is solved by the relation,

$$- 2e^{-2\phi}\dot{\phi} - \kappa\dot{\phi} + \frac{\kappa}{2}\frac{\dot{a}}{a} = 0 \,. \tag{22}$$

Defining $\xi \equiv e^{-2\phi}$, the above condition yields

$$\dot{\xi} + \frac{\kappa}{2}\frac{\dot{\xi}}{\xi} + \frac{\kappa}{2}\frac{\dot{a}}{a} = 0 \quad, \tag{23}$$

which gives:

$$a = \frac{1}{\xi}e^{-\frac{2}{\kappa}\xi} . \tag{24}$$

The two equations of motion (9) and (10) reduce to the following differential equations in ξ and its derivatives,

$$\left(1 + \frac{\kappa}{4\xi}\right)\ddot{\xi} - \left(\frac{3}{2\xi} + \frac{2}{\kappa} + \frac{\kappa}{2\xi^2}\right)\dot{\xi}^2 = 0 \quad, \tag{25}$$

$$\left(\frac{2}{\kappa} + \frac{1}{\xi}\right)\left[\left(1 + \frac{\kappa}{4\xi}\right)\ddot{\xi} - \left(\frac{3}{2\xi} + \frac{2}{\kappa} + \frac{\kappa}{2\xi^2}\right)\dot{\xi}^2\right] = 0 . \tag{26}$$

Notice that the prefactor in the second equation of motion, if set to zero, results in a constant ξ (and therefore a constant ϕ). Discarding this trivial solution, we finally have only one differential equation to solve, namely, Eq. (26).

The solution to this equation gives us τ as a function of ξ :

$$\tau = \kappa \left[-2Ei(-2\frac{\xi}{\kappa}) - \kappa\frac{e^{-2\frac{\xi}{\kappa}}}{\xi}\right] \tag{27}$$

where $Ei(-\xi)$ is the exponential integral function [1].

Figures 2(a) and 2(b) illustrate the two functions $\tau(\xi)$ and $a(\tau)$, respectively. Note that only negative values of τ are allowed because otherwise ξ is negative, which implies a negative coupling to gravity. The solution for $a(\tau)$ has a singularity at $\tau = 0$ and there is no physically meaningful solution in the $\tau > 0$ region. It can be verified that the second branch solution for a, given by (24) and (27), is related to the first branch solution (17) through the duality transformation (13).

In Rey's model, the solution corresponding to the second branch is not expressible in terms of known functions, in the region of positive dilaton coupling.

_____

[1] We follow the definition, $Ei(-x) = -\int_x^\infty \frac{e^{-x}}{x}dx.$

A comment about the *energy conditions* in these geometries. In $1+1$ dimensions, the Raychaudhuri equation for timelike geodesic congruences is given as [14,15]:

$$\frac{d\theta}{d\lambda} + \theta^2 = \frac{1}{2}R\,.\tag{28}$$

The condition for focusing of timelike geodesics therefore reduces to $R \leq 0$. For the cosmological metrics under consideration $R = 2\ddot{a}/a$. It is therefore clear that geodesics will not focus in this geometry within a finite value of λ because the convergence condition is violated. In fact the convergence condition can only be satisfied at future infinity where the scale factor is proportional to τ. The fact that quantum effects are essentially responsible in allowing the existence of such a geometry makes the violation of the convergence condition only more plausible. Recall that in four-dimensional (4D) scenarios the violation of the energy conditions is often attributed to 'quantum' matter [13].

We now discuss the distribution of quantum matter and the status of the energy conditions in our solutions. As mentioned above, in our model the total matter stress tensor is determined solely by the one-loop contribution $\langle T_{\mu\nu}\rangle$, which in turn depends on the choice of the matter state as defined in (7). Since it is the first branch that gives an interesting cosmological evolution, we calculate $\langle T_{\mu\nu}\rangle$ only for this case. The only nonvanishing component turns out to be $\langle T_{\tau\tau}\rangle = \kappa/4\left[3\ddot{a}/a - 8\left(\dot{a}/a\right)^2\right]$. For the first branch solution (17), this gives

$$\langle T_{\tau\tau}\rangle = -\frac{\kappa\alpha^2}{4[(\alpha\tau+\beta)^2+\kappa]^{3/2}}\left[3(\alpha\tau+\beta)+5\sqrt{(\alpha\tau+\beta)^2+\kappa}\right]\,,\tag{29}$$

which is always nonpositive. To physically interpret the matter state, note that the above expression vanishes asymptotically as $\tau \to \pm\infty$. Since for this branch the scalar curvature also vanishes in these regimes, it implies that matter state is the 'in' or 'out' Minkowski vacuum.

The Weak Energy Condition (WEC) is violated in this branch since the average density of matter is negative. However, unlike in 4D Einstein gravity, in this 2D model the equations of motion show that the Ricci tensor is not exclusively determined by the matter stress tensor.

Thus, just the fact that $\langle T_{\tau\tau} \rangle$ is nonnegative does not necessarily guarantee defocusing of the timelike geodesics and, hence, the cure to the graceful exit problem. Nevertheless, as shown above, explicit calculation of the Ricci scalar for the solution (17) does bear out such a behavior of geodesics in these spacetimes. The effect of violation of energy conditions on the solution to the graceful exit problem in 4D is studied in Ref. [16].

We conclude the paper with a few remarks. First, it is important to note that in Rey's solution to the graceful exit problem the barrier $N < 24$ is not merely a restriction on the permissible number of matter fields in that theory. More important, it violates the large N approximation [1], which is assumed at the outset when only the one-loop correction is retained in the effective action after expanding it in powers of the Planck's constant. In fact, large N approximation is used to drop the higher order corrections as well as corrections due to ghost contributions. Therefore, in order to get a consistent model one should allow for N to be large in its solutions.

Second, here we have concentrated only on 2D cosmological models. However, realistically it is important to address the graceful exit problem in 4D. The no-go theorems have shown that modifying the 4D tree level action by adding realistic dilatonic potentials [17], axions [18] or even tachyons [19] does not cure this problem. Calculations within a quantum cosmology setting do indicate the possibility of an exit [20]. Also the result that an exit is possible only if the weak energy condition is violated indicate that the graceful exit problem might be a classical feature that can be cured only by including quantum corrections. This urges one to study solutions of the 4D effective action (see, eg., [4]) expanded to the next higher order in the coupling, which is a nontrivial problem. An alternative is to perform a semiclassical treatment by quantizing only the matter fields and retaining only one-loop terms by invoking the large N approximation. This option is not trivial either because of the presence of higher derivative terms. Thus, for a beginning, a tractable calculation that includes quantum effects can be outlined as follows. In the spirit of the minisuperspace program, one imposes isotropy at the level of the vacuum 4D tree action and performs an S-wave reduction (by integrating over the angular coordinates). The resulting 2D action

11

is the CGHS action. Thus the semiclassical solutions of the CGHS action, which we have discussed in this paper, illustrate the semiclassical behavior of the S-wave sector of the 4D theory. This suggests that perhaps the 4D effective action expanded to higher orders may be devoid of the graceful exit problem.

Finally, note that the Polyakov-Liouville term in the one-loop corrected action breaks both conformal invariance and SFD. Since the 4D effective action preserves SFD at higher orders in the perturbative expansion [21], one might question if our results, which break SFD, have any bearing on the 4D quantum-corrected solutions. We do not have a completely satisfactory answer to this question, except to point out that the large N approximation allows us to neglect the quantum corrections due to the dilaton, metric, and ghosts. Making such an approximation on the S-wave sector of the 4D effective action expanded to first order will presumably result in a theory without SFD. It is in this limit that our 2D solutions might make a correspondence with their 4D counterparts.

REFERENCES

[1] C. G. Callan, S. Giddings, J. Harvey, A. Strominger, Phys. Rev. D**45**, 1005 (1992).

[2] F. D. Mazzitelli, J. G. Russo, Phys. Rev. D**47**, 4490 (1993).

[3] S.-J. Rey, *Phys. Rev. Lett.*, **77**, 1929 (1996).

[4] G. Veneziano, in *String Gravity and Physics at the Planck Energy Scale*, eds. N. Sanchez and A. Zichichi (Kluwer Academic Publishers, Netherlands), p. 285 (1996); M. Gasperini, *ibid.*; S.-J. Rey, *Recent progress in string inflationary cosmology*, Report No. SNUTP-96-087 (hep-th/9609115).

[5] N. Sanchez, *Nucl. Phys.*, **B266**, 487 (1986).

[6] M. Gasperini, G. Veneziano, Phys. Lett. **B387**, 715 (1996).

[7] W. T. Kim and M. Yoon, *Quantization of dilaton cosmology in two dimensions*, hep-th/9704115.

[8] S. Bose, *Solving the graceful exit problem in superstring cosmology*, hep-th/9704175.

[9] J. G. Russo, L. Susskind, L. Thorlacius, Phys. Rev. D**46**, 3444 (1992).

[10] S. Odintsov and I. Shapiro, Phys. Lett. **B263**, 181 (1991); Int. J. Mod. Phys. D**1**,571 (1993); J. G. Russo, A. Tseytlin, Nucl. Phys. **B382**, 259 (1992).

[11] S. Bose, L. Parker, and Y. Peleg, Phys. Rev. D**52**, 3512 (1995).

[12] P. C. W. Davies, S. A. Fulling, W. G. Unruh, Phys. Rev. D**13**, 2720 (1976).

[13] N. D. Birrell and P. C. W. Davies, *Quantum Fields in Curved Space*, (Cambridge University Press, Cambridge, England, 1982).

[14] K. C. K. Chan and R. B. Mann, Class. Quant. Grav. **10**, 913 (1993).

[15] S. Kar, Phys. Rev. D**55** 4872 (1997).

[16] R. Brustein and R. Madden, *Graceful exit and energy conditions in string cosmology*, Report No. BGU-PH-97-06 (hep-th/9702043).

[17] R. Brustein and G. Veneziano, Phys. Lett. **B329**, 429 (1994).

[18] N. Kaloper, R. Madden, and K. A. Olive, Nucl. Phys. **B452**, 1995 (1995); Phys. Lett. **B371** 34 (1996).

[19] S. Mahapatra, K. Maharana, and L. P. Singh, *A tachyonic extension of the stringy no-go theorem*, Report No. UUPHY-95-10 (hep-th/9503216).

[20] M. Gasperini, J. Maharana, and G. Veneziano, Report Nos. CERN-TH/96-32 and IP-BBSR/96-9 (hep-th/9602087); J. Maharana, S. Mukherji, S. Panda, Report Nos. IP-BBSR/96-67, CTP-TAMU-63/96, and MRI-PHYS/96/34 (hep-th/9701115).

[21] K. A. Meissner, Phys. Lett. **B392**, 298 (1997).

FIGURE CAPTIONS

Fig. 1 The scale factor $a(\tau)$ versus τ as given in Eqn. (17) (+ sign). Here $\alpha = 1$, $\beta = 0$ and $\kappa = 2$.

Fig. 2 (a) τ as a function of $\xi = e^{-2\phi}$. Here, $\kappa = 2$.

Fig. 2 (b) The scale factor $a(\tau)$ versus τ as given (in parametric form) by Eqns. (24) and (27). Here, $\kappa = 2$.
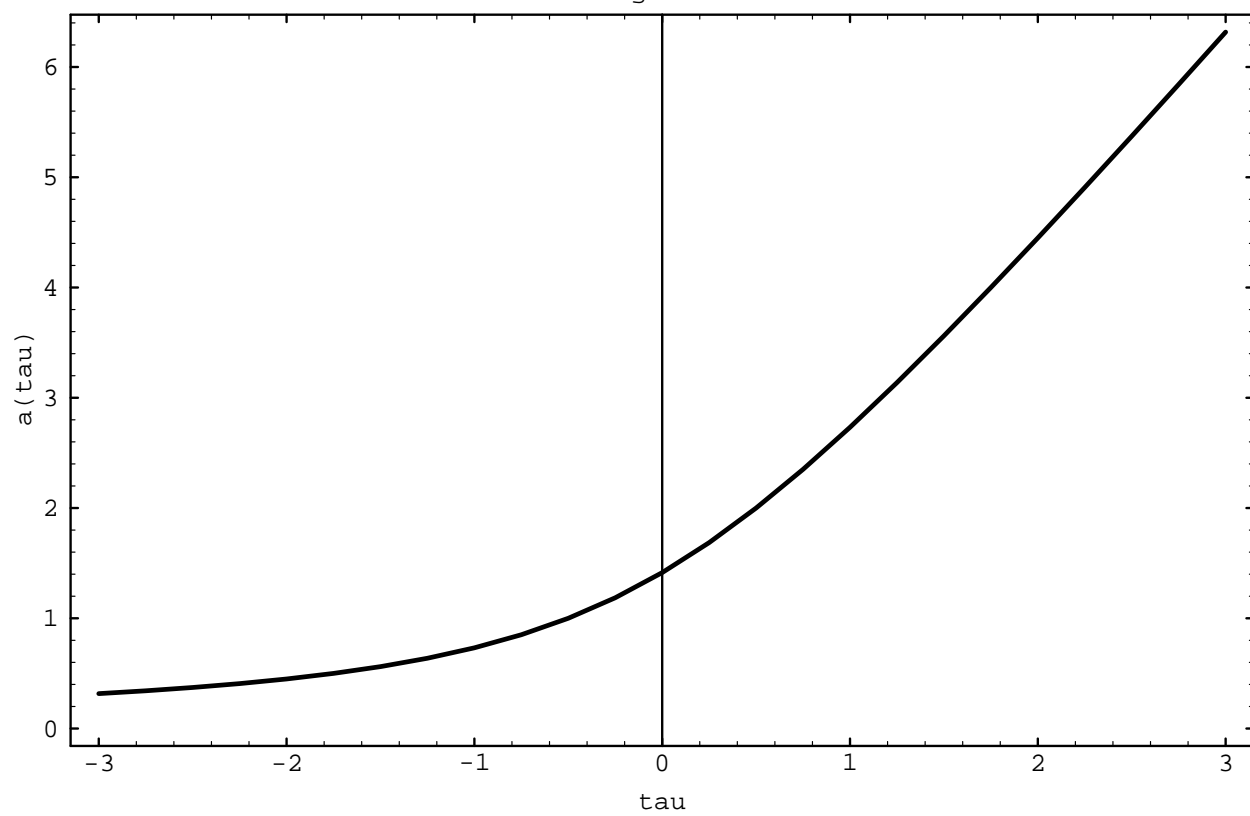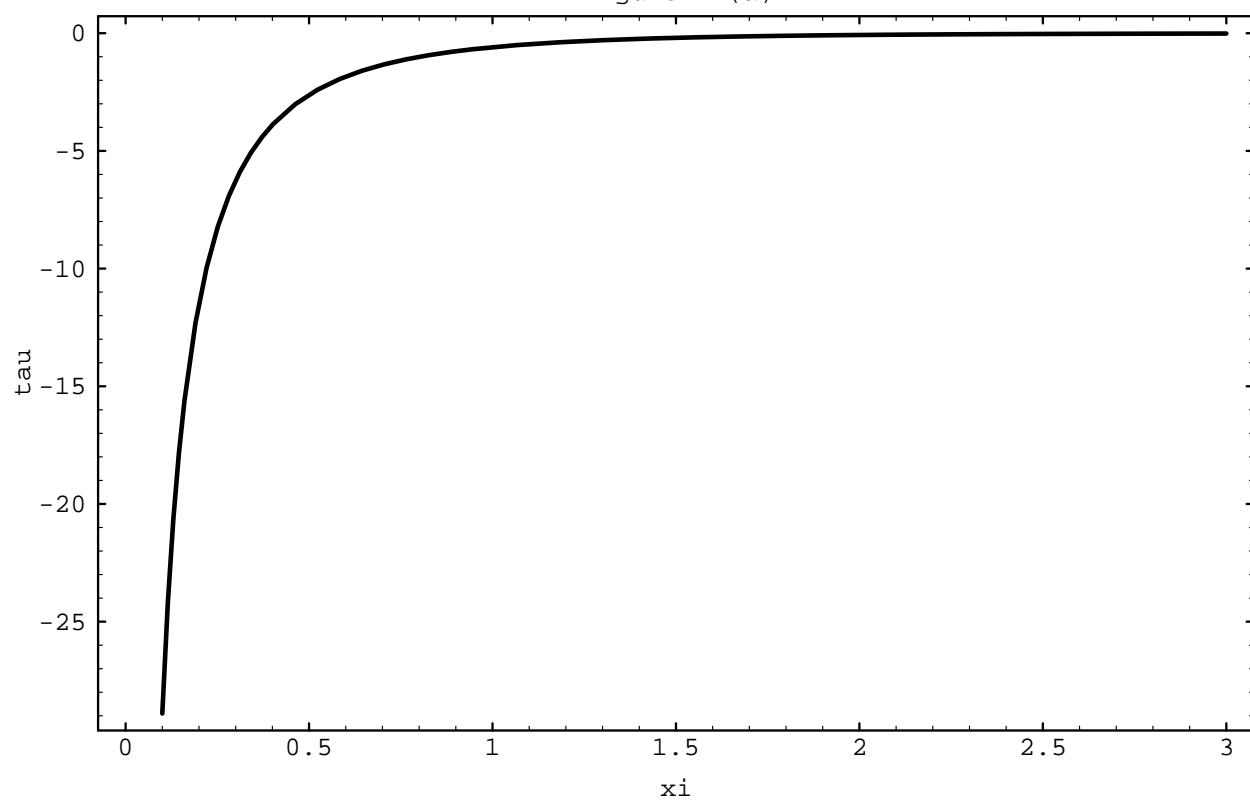
Figure 1

Figure 2 (a)

Figure 2(b)